**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

A.0.3875  
B. 0.2676  
C. 0.5  
D. 0.6987

Ans: We have a normal distribution with = 45 and = 8.0. Let X be the amount of time it takes to complete the repair on a customer's car. To finish in one hour you must have X d" 50 so the question is to find Pr(X > 50).

Pr(X > 50) = 1 - Pr(X d" 50).

>Z = (X - µ)/ = (X - 45)/8.0

Thus the question can be answered by using the normal table to find

Pr(X d" 50) = Pr(Z d" (50 - 45)/8.0) = Pr(Z d" 0.625)=73.4%

Probability that the service manager will not meet his demand will be = 100-73.4 = 26.6% or 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans - We have a normal distribution with = 38 and = 6. Let X be the number of employees. So according to question

a)Probabilty of employees greater than age of 44= Pr(X>44)

Pr(X > 44) = 1 - Pr(X d" 44).

Z = (X -µ )/ = (X - 38)/6

Thus the question can be answered by using the normal table to find

Pr(X d" 44) = Pr(Z d" (44 - 38)/6) = Pr(Z d" 1)=84.1345%

Probabilty that the employee will be greater than age of 44 = 100-84.1345=15.86%

So the probability of number of employees between 38-44 years of age = Pr(X<44)-0.5=84.1345-0.5= 34.1345%

Therefore the statement that More employees at the processing center are older than 44 than between 38 and 44 is TRUE.

b) Probabilty of employees less than age of 30 = Pr(X<30).

Z = (X - )/ = (30 - 38)/6

Thus the question can be answered by using the normal table to find

Pr(X d" 30) = Pr(Z d" (30 - 38)/6) = Pr(Z d" -1.333)=9.12%

So the number of employees with probability 0.912 of them being under age 30 = 0.0912\*400=36.48( or 36 employees).

Therefore the statement B of the question is also TRUE.

3.If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans:

The Normal Distribution has its link with the Central Limit Theorem, which states that ‘Any large sum of independent identically distribution random variables are approximately Normal then

(X1 + X2) and (2X1) tends to have Normal distribution only If X1 and X2 are i.i.d and n is Large.

The Difference between 2X1 and (X1 + X2) is the magnitude they hold of two different sample subsets (X1 and X2) from the same source(population).

X1 and X2 can be a different subset of a sample from a similar source (population) but

If X1 ~ N(μ, σ2) then, 2 X1 ~ N(2 μ, 4 σ2 )

If X1 ~ N(μ, σ2) and X2 ~ N(μ, σ2) are iid normal random variables then

(X1 + X2)~N(μ+ μ, σ2+ σ2)~(2 μ, 2 σ2)

Hence, 2X1 – (X1+X2) ~(2 μ – 2 μ, 4 σ2 + 2σ2 )

The distribution remains the same for every sample subset of similar source, it tends to fall under Normal distribution and slight deviations in parameters.

The Normal distribution has two parameters, the mean, µ, and the variance, σ2.

µ and σ2satisfy −∞ < µ < ∞, σ2> 0. We write X ∼ Normal (µ, σ2) or X ∼ N(µ, σ2 ).

4.Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

1. 90.5, 105.9
2. 80.2, 119.8
3. 22, 78
4. 48.5, 151.5
5. 90.1, 109.9

Ans. Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.

The Probability of getting value between a and b should be 0.99.

So the Probability of going wrong, or the Probability outside the a and b area is 0.01 (ie. 1-0.99).

The Probability towards left from a = -0.005 (ie. 0.01/2).

The Probability towards right from b = +0.005 (ie. 0.01/2).

So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

Z=(X- ¼ ) / Ã

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \* Ã + ¼ = X

Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4

Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6

So, option D is correct.

5.Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: Rupee ranges in between [9.9 to 98.1] Crore Rupees, 95% of the time for the Annual Profit of the Company.

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans: The 5TH Percentile of profit for the company is 17 Crore Rupees

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: The Division #2 (Profit2 ~ N(7, 42) ) has a larger probability of making a loss in a given year